VistaVision/Duo/SC Series
Application Note BD-05

DERIVATION OF SIGNAL TO NOISE RATIO (SNR)

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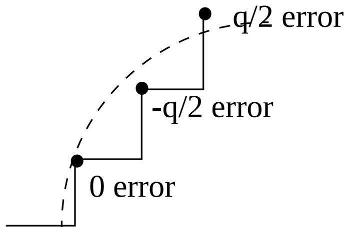
When working as test engineers in the mixed signal world we often encounter the formula for signal-to-noise ratio (SNR), usually written as :

$$\mathbf{SNR = 6.02n + 1.76}$$

where n is the effective number of bits (ENOB). We tend to take this formula at face value, but it may be of interest to understand how it is derived.

If we overlay a picture of acontinuous signal with a picture showing its digitized equivalent, it will look like Figure 1 below. If we then graph the expected value of the error, E(x), we get the function shown in Figure 2.



q = quantization value (1 LSB)

Figure 1 - ADC quantization

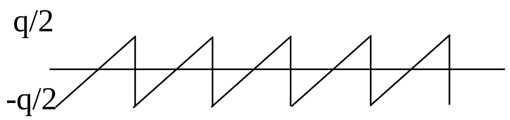


Figure 2 - Quantization error

From calculus, we can find the RMS error for the function E(x) as

$$\delta^2 = \int_{-\frac{q}{2}}^{\frac{q}{2}} ((x^2 \cdot (1/q)) \cdot dx) \; = \; (1/q)(q^3/(3 \cdot 8) + q^3/(3 \cdot 8)) = \; (q^2/12)$$

RMS Error

$$\delta \; = \; q/(\sqrt{12})$$

Figure 3 - RMS Error for an ADC with LSB value *q*

Signal-to-noise ratio (SNR), can be viewed as the magnitude of the signal divided by the magnitude of any error, or noise. In a perfect ADC, the only error will be that introduced through quantization. The SNR value is often given in decibels, because the ratio can be so large.

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RMS signal = $A/(\sqrt{2})$ where A is the peak amplitude

For an n-bit ADC, peak value A = $q(2^{n-1})$

Therefore, A_{rms} = $q\ (2^{n-1})\ /\sqrt{2}$

Figure 4 - RMS Signal for an *n*-bit ADC

We can then develop the SNR as the ratio of the RMS value of the fundamental signal to the RMS value of the quantization error function:

SNR = RMS Signal / RMS Error

$$\begin{aligned} &= \; 20\log\left(q\frac{2^{n-1}}{\sqrt{2}}/(q\sqrt{12})\right) \\ &= \; 20\log 2^{n-1}\sqrt{6} \\ &= \; 20n\log 2 + 20\log\sqrt{6}/2 \\ &= \; 6.02n + 1.76 \end{aligned}$$

Figure 5 - Signal-to-Noise Ratio (SNR) in decibels

EXAMPLE:

Consider an 8-bit ADC with an output range of 4V. Its LSB is worth $4/2^{(n-1)}$ = 31.25mV. The ideal SNR of this ADC will be 6.02(8) + 1.76 = 49.92 dB.

If we measure the ADC on the Duo test system and measure an actual SNR of 40dB, then we can calculate the Effective Number of Bits (ENOB) = (40 - 1.76) / 6.02 = 6.35 bits. In other words, if we had an ideal 6.35 biy ADC we would get the same performance as this actual ADC. Or, our 8 bit ADC has 6.35 *useable* bits (bits which give us actual signal strength, or dynamic range, above the noise level).

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